Problem 4.38

Quarks carry spin 1/2. Three quarks bind together to make a **baryon** (such as the proton or neutron); two quarks (or more precisely a quark and an antiquark) bind together to make a **meson** (such as the pion or the kaon). Assume the quarks are in the ground state (so the *orbital* angular momentum is zero).

(a) What spins are possible for baryons?

(b) What spins are possible for mesons?

Solution

According to page 178, the rule is that if spin s_1 and spin s_2 are combined, the possible resultant spins go from $|s_1 - s_2|$ to $s_1 + s_2$ in integer steps.

Part (a)

Suppose three quarks in the ground state bind together to make a baryon. Calculate $|s_1 - s_2|$ and $s_1 + s_2$ for the combination of the first two quarks.

$$\left|\frac{1}{2} - \frac{1}{2}\right| = 0 \quad \text{and} \quad \frac{1}{2} + \frac{1}{2} = 1$$

Now evaluate $|s_1 - s_2|$ and $s_1 + s_2$ for the combination of the first two quarks with the third.

$$\left.\begin{aligned} \left|0 - \frac{1}{2}\right| &= \frac{1}{2} \\ 0 + \frac{1}{2} &= \frac{1}{2} \end{aligned}\right\} \quad \text{and} \quad \left.\begin{aligned} \left|1 - \frac{1}{2}\right| &= \frac{1}{2} \\ 1 + \frac{1}{2} &= \frac{3}{2} \end{aligned}\right\}$$

Therefore, the possible spins of a baryon are 1/2 and 3/2, assuming the constituent quarks are in the ground state.

Part (b)

Suppose two quarks in the ground state bind together to make a meson. Calculate $|s_1 - s_2|$ and $s_1 + s_2$ for the combination of these two quarks.

$$\left|\frac{1}{2} - \frac{1}{2}\right| = 0 \quad \text{and} \quad \frac{1}{2} + \frac{1}{2} = 1$$

Therefore, the possible spins of a meson are 0 and 1, assuming the constituent quarks are in the ground state.